Is This Function a Bijection?

1a.) Define a function from N (Natural Numbers) to Z (Integers) that is both one-to-one and onto.

In order for this function to be one-to-one and onto each of the natural numbers needs to correspond to only one integer. Since the integers cover negative values, while the natural numbers don’t, in order for the natural numbers to be one-to-one with the integers a formula must be created.

f: N🡪Z given by f(z) = |z|

This notation states that every integer, or Z value, that goes in the function must return its absolute value. This way the natural numbers are one-to-one with the integers. By creating the formula that takes the absolute value of the integers a bijection is created. This is because the function is now both one-to-one and onto.

1b.) Define a function from R (Real) to R (Real) that is a bijection but is not a linear function.

In order to fulfil the requirements of the question a function for R to R must be created that is not a straight line, but is a bijection.

f: R🡪R given by f(r) = |r^2|

The above function which takes a real number and squares its value and then takes the absolute value of that number is one-to-one and it is not linear. Adding the absolute value to the function ensures that the function is one to one. Without the absolute value bars the function would be a parabola, which fails the horizontal line test and would not be one-to-one.